Rupture of Rubber. VI. Further Experiments on the Tear Criterion

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1. INTRODUCTION

In previous papers (I to V of this series),[1-5] a tear criterion for rubbers has been proposed based on an energy balance approach. This equates the energy required to form new surfaces (the tearing energy) with the loss of elastic strain energy in the test piece. The tearing energy T is assumed to be characteristic of the material and so independent of the overall shape of the test piece. It is thus the fundamental property controlling tear behavior. The correctness of this approach was investigated by making tear measurements on test pieces of different shapes but of the same material and examining the constancy of the T values obtained.[1,5] The results weie consistent with the theory but not wholly conclusive, due primarily to the particular tearing behavior of the materials used (natural rubber gum vulcanizates). Another limitation was that accurate T values could be obtained only if they could be calculated directly from the measured tearing forces or elongations, and the required relationships were known for only two types of test piece. Clearly, the more test pieces available for comparison and the more they differ from each other in shape, the more stringent the test of the basic theory. In the present paper a third test piece is described, the necessary theory given, and experimental results presented on the three test pieces. By comparing the results from these test pieces, which are of widely different shapes, a critical test of the theory is possible.

The choice of the experimental material is influenced by several factors. Previous measurements have been made on natural rubber gum compounds,[1] which have the advantage of possessing excellent elastic properties but whose rupture characteristics are such that tearing occurs at a critical load. In contrast, a gum GR-S tears more or less steadily at a rate depending on the load,[3] a characteristic which is experimentally advantageous for the particular test pieces described here. It was therefore used in this investigation.

2. COMPARISON OF THE "PURE SHEAR" AND "SIMPLE EXTENSION" TEAR TEST PIECES

The proposed tear criterion[1] states that the amount of work necessary to make a cut of length c grow by a small amount Δc, thus creating an area $t\Delta c$ of new surface, is equal to $Tt\Delta c$, where T is a constant of the material and independent of the overall shape of the test piece and t is the thickness. If this cut growth occurs without the applied forces on the test piece moving, the energy is derived only from the elastic strain energy in the test piece. The criterion can thus be written as

$$-1/t(\partial W/\partial c)_l = T \qquad (2.1)$$

where W is the total strain energy and the subscript l indicates that the differentiation is carried out with the applied forces not moving and so doing no work.

In part I the quantity $1/t(\partial W/\partial c)_l$ was calculated for two test pieces, called "simple extension" and "pure shear" (Figs. 1 and 2) in terms of their dimensions, the elastic properties of the material,

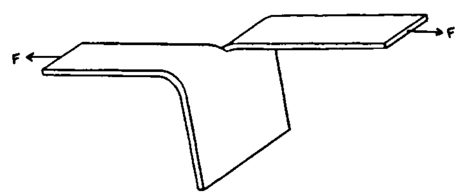

Fig. 1. Simple extension tear test piece.

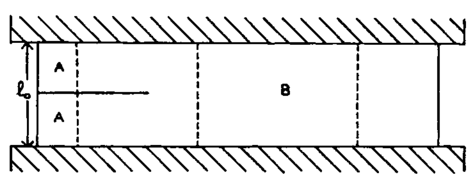

Fig. 2. Pure shear tear test piece.

and the applied forces or strains. The dimensions of the simple extension test piece are such that there are regions in simple extension in the arms and a substantially unstrained region beyond the tip of the cut. The calculation for this test piece gives [eq. (6.8 in I)]:

$$-(\partial W/\partial c)_l = 2\lambda F - E_e A_0 \qquad (2.2)$$

where F is the applied force, λ the extension ratio in the arms, E_e the elastically stored energy per unit volume in the arms, and A_0 the unstrained cross sectional area of the test piece. The pure shear test piece must have regions A unstrained and a region B in pure shear. The calculation for this test piece gives [eq. (6.10) in I]:

$$-1/t(\partial W/\partial c)_l = E_s l_0 \qquad (2.3)$$

where E_s is the elastically stored energy per unit volume in the pure shear region and l_0 is the unstrained length of the test piece.

In part I, experiments were done to test eqs. (2.1)-(2.3) using natural rubber pure gum vulcanizates. A test piece of this material pulled slowly at room temperature tears catastrophically, giving a single value of T. A GR-S gum rubber, however, tears continuously,[3] the rate of tearing depending strongly on the applied load. In this case, therefore, T has no unique value but is a function of the rate of tearing r, and comparisons have to be made over a range of r and T.

The same GR-S vulcanizate was used for all the measurements reported in this paper, the formulation being given in Table I. The dependence of T on r for a simple extension test piece was found by the method[6] described in part III, *viz.*, by pulling the test piece at known rates of clamp separation and measuring the average tearing forces developed. The T values were calculated from the forces by eq. (2.2), and the results are shown as the full curve in Figure 3.

TABLE I

Component	Parts
Polysar S	100
Zinc oxide	5
Stearic acid	2
Nonox H.F.N.	1
Santocure	1
Sulfur	1.75

Cure: 145°C. for 50 min.

The corresponding measurements on the pure shear test piece were made as follows. The apparatus for extending the test piece was similar to that described in II, consisting of two parallel clamps, about 30 cm. long, the separation of which could be varied by screws. With l_0 about 5 cm., the strain distribution conditions for this test piece were satisfied for 5 cm. $< c <$ 15 cm., with wider tolerances for smaller l_0 values. The test pieces were strained to a suitable extent and λ_s calculated from the measured separations of marks on the rubber. The incision was then made, and during the subsequent tearing the passage of the tip past reference marks on the rubber was timed, giving r. To find T, E_s must be known in terms of λ_s. This relation was found by integrating graphically under the pure shear stress–strain curve obtained by the method of Rivlin and Saunders.[7] Equations (2.3) and (2.1) then gave T. Test pieces of lengths 1.5, 3.0, and 5.0 cm. were used, and the results are shown

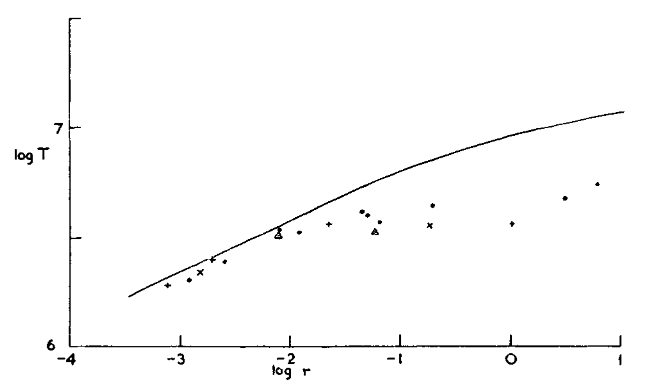

Fig. 3. Comparison of results from the simple extension and pure shear test pieces: (——) simple extension test piece with constant rate of grip separation: (●) simple extension test piece with use of constant force apparatus; pure shear test piece for (△) l_0 = 5.0 cm.; (+) l_0 = 3.0 cm.; (×) l_0 = 1.5 cm.

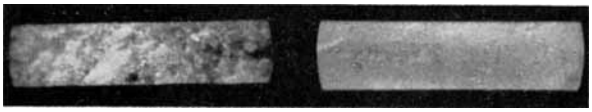

Fig. 4. Photographs of torn surfaces (left) torn at low rates; (right) torn at high rates.

in Figure 3, where they may be compared with those obtained from the simple extension test piece.

The results show (*1*) that the pure shear measurements are self-consistent, the results for test pieces of different lengths agreeing with each other; (*2*) that there is approximate agreement with the simple extension measurements below rates of about 10^{-2} cm./sec.; and (*3*) that above this critical rate r_c, however, there is marked disagreement between the behavior of these two test pieces, the pure shear test piece giving much higher rates of tear. The manner of tearing of the pure shear test piece undergoes a fairly abrupt change at r_c. At lower rates, r fluctuates somewhat, but it is rarely more than about three times the average. Above r_c, the rate fluctuates between low values ($\sim 10^{-2}$ cm./sec.) and much higher ones ($\sim$1–10 cm./sec.), and the tearing has a "stick–slip" character. The average rate observed in this region is governed by the relative times the tear is in the "stick" or "slip" states. The appearance of the torn surfaces correlates with these rate changes. Below r_c the surfaces are rough and irregular, but above it in the "slip" regions they are much smoother and reflect light specularly; in "stick" regions they are rough. The photographs in Figure 4 illustrate this.

The large fluctuation in tearing rate observed with the pure shear test piece above r_c suggests that the discrepancy between this and the simple extension test piece may arise from the different averaging procedures. The method of measurement with the latter test piece is to separate the arms at a known constant rate and measure the resulting force. The latter is not constant, fluctuating by about $\pm 20\%$ in this region; also the actual rate of cut growth varies considerably due to the compliance of the test piece and the force measuring device. Thus both r and T are averages. In the pure shear case, T is governed by the strain in the test piece and, apart from the slight inertial effect of the rubber, should be constant with time and independent of r. This measurement therefore gives an average at constant T in contrast with the simple extension case, in which both T and r fluctuate.

To find if this explanation of the discrepancy is correct, measurements were made on the simple extension test piece under conditions nearer to constant T, i.e., constant applied force F. A dead load applied by a weight is not satisfactory if stick-slip behavior is expected, as the accelerations may be comparable with the gravitational acceleration. A device, shown in Figure 5, was therefore constructed to produce a substantially constant force with little associated inertia. An aluminum drum A, about 0.5 cm. in diameter, was connected to rubber strips about 30 cm. long, and a thin nylon monofilament wound on it. This was connected to one arm of the test piece C, the other arm being joined to a cantilever spring S. This carried a mirror M, so that, by use of a lamp and scale, the force could be found. The rubber strips were twisted by devices at their ends (not shown) and so transmitted a force to the test piece. The inertia associated with the drum and the elastic strips was small, being comparable to that of the test piece itself. As the test piece tore, the strips untwisted and the force fell, but this change was kept to within 5% of the mean.

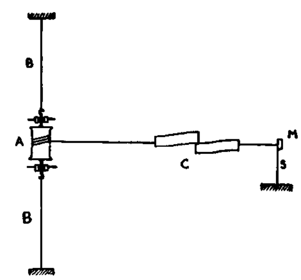

Fig. 5. Constant force apparatus.

Test pieces with dimensions of about 10 × 3 × 0.1 cm. were used. The results, given in Figure 3, show that the discrepancy with the pure shear measurements is now greatly reduced. The remaining differences may be due to the appreciable inertia still present, mostly in the test piece itself, as the force indicated by the spring still showed noticeable fluctuations. The substantial improvement obtained does, however, indicate that the original discrepancy was due primarily to the suggested cause and not to a basic deficiency in the theory.

3. FURTHER RESULTS ON THE PURE SHEAR AND SIMPLE EXTENSION TEST PIECES

In view of the tear behavior of GR-S at tearing rates greater than 10^{-2} cm/sec., accurate comparisons of different test pieces are best carried out below this rate. The results of Section 2 show

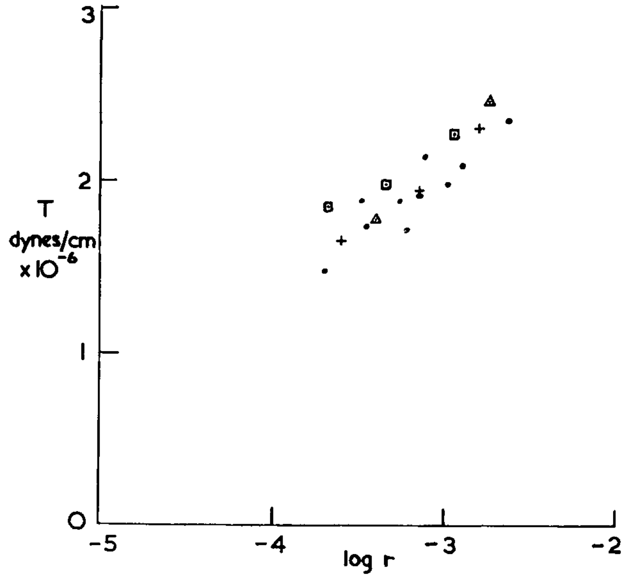

Fig. 6. Comparison of results from simple extension and pure shear test pieces at low tearing rates: (●) simple extension; pure shear for (□) l_0 = 5.0 cm., (△) l_0 = 3.0 cm., (+) l_0 = 2.0 cm.

approximate agreement in this range, but more results are needed to establish this definitely.

The present experiments were all done on test pieces cut from the same sheet of rubber (prepared as before) to minimize variations. The simple extension measurements were made by hanging weights on the test pieces and observing the rate of movement of the arms. The pure shear measurements were carried out as described in Section 2 on three different sizes of test piece.

The experiments were done at room temperature, which was 20 ± 2°C. This variation was found to have a perceptible effect, so subsidiary measurements were made at 17 and 25°C. which indicated a change in T with temperature of 0.075 kg./cm./°C. for a fixed rate of tearing. The results were then corrected to 20°C.

The results for both the simple extension and pure shear test pieces are shown in Figure 6. The agreement between these two test pieces and between the pure shear test pieces of different sizes is very satisfactory, all giving results consistent with the same T-r relation.

4. CALCULATIONS ON THE SPLIT TEAR TEST PIECE

In this section a new tear test piece is described and its elastic properties calculated so that the tear criterion may be applied

The test piece is shown in Figure 7. It is deformed by two pairs of forces F_1 and F_2, and its dimensions are such that there are regions A and B in simple extension. The force S in region A makes an angle θ with the test piece axis, and the extension ratios in regions A and B are λ_1 and λ_2, respectively.

For the tear criterion to be applied, $(\partial W/\partial c)_{l,y}$ must be known in terms of directly measurable quantities.

We have

$$dW = (\partial W/\partial c)_{l,y}dc + (\partial W/\partial l)_{c,y}dl + (\partial W/\partial y)_{c,l}dy \quad (4.1)$$

and

$$F_1 = (\partial W/\partial l)_{c,y} \qquad F_2 = (\partial W/\partial y)_{c,l} \quad (4.2)$$

Hence, by substitution,

$$(\partial W/\partial c)_{F_1,F_2} = (\partial W/\partial c)_{l,y} + F_1(\partial l/\partial c)_{F_1,F_2} + F_2(\partial y/\partial c)_{F_1,F_2} \quad (4.3)$$

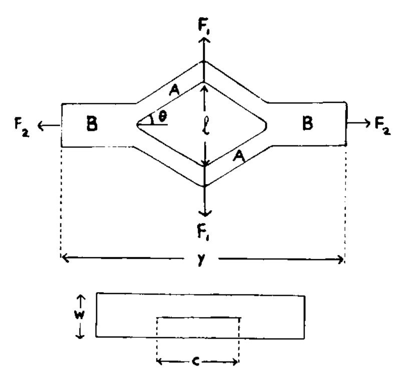

Fig. 7. Split tear test piece.

As regions A and B are in simple extension, the complex strains around the points of applications of F_1 and around the ends of the cut will not vary with c, provided F_1 and F_2 are constant. If c increases by Δc, for example, the regions at the ends of the cut will move outwards by Δc but will still have the same total stored energy, the net effect being for regions A to grow at the expense of regions B. Under these conditions

$$F_1 = 2S \sin \theta \quad (4.4)$$

$$F_2 = 2S \cos \theta$$

$$(\partial l/\partial c)_{F_1,F_2} = \lambda_1 \sin \theta \quad (4.5)$$

$$(\partial y/\partial c)_{F_1,F_2} = \lambda_1 \cos \theta - \lambda_2 \quad (4.6)$$

$$(\partial W/\partial c)_{F_1,F_2} = wt(E_1 - E_2) \quad (4.7)$$

In eq. (4.7), E_1 and E_2 are the stored energies per unit volume in the regions A and B, respectively. They can be found from λ_1 and λ_2 by using the measured stress-strain relation for the rubber. Substituting eqs. (4.5)-(4.7) in eq. (4.3), yields

$$-(\partial W/\partial c)_{l,y} = F_1\lambda_1 \sin \theta + F_2(\lambda, \cos \theta - \lambda_2) - wt(E_1 - E_2) \quad (4.8)$$

and, from eq. (4.4) we obtain

$$\tan \theta = F_1/F_2 \quad (4.9)$$

Thus, eqs. (4.4), (4.8), and (4.9), together with the stress-strain curve, give $(\partial W/\partial c)_{l,y}$ in terms of F_1 and F_2.

If $F_2 = 0$, the test piece becomes equivalent to two simple extension test pieces joined together, and eq. (4.8) reduces to eq. (2.2), apart from the factor of 2. When F_2 is comparable with F_1, the behavior of the test pieces is very different, and it is in this range that comparisons will be made.

A very useful approximation to eq. (4.8) can be made when λ_1 is not much greater than λ_2. Putting $\lambda_1 = \lambda_2 + \Delta\lambda$, we have approximately

$$wt(E_1 - E_2) = \Delta\lambda(F_2 + 2S)/2 \quad (4.10)$$

From eq. (4.4) we have

$$2S = \sqrt{F_1^2 + F_2^2} \quad (4.11)$$

and, substituting eqs. (4.9), (4.10), and (4.11) in eq. (4.8), we obtain

$$-(\partial W/\partial c)_{l,y} = \bar{\lambda}\left(\sqrt{F_1^2 + F_2^2} - F_2\right) \quad (4.12)$$

where $\bar{\lambda} = {}^1/_2(\lambda_1 + \lambda_2)$. Comparison of the exact relation, eq. (4.8), with eq. (4.12) for a rubber obeying the statistical theory indicates that eq. (4.12) is a good approximation; for example, with $\lambda_1 = 3.0$ and $\lambda_2 = 1.5$, it is in error by less than 2%. In the experiments on this test piece the error from this cause is even less.

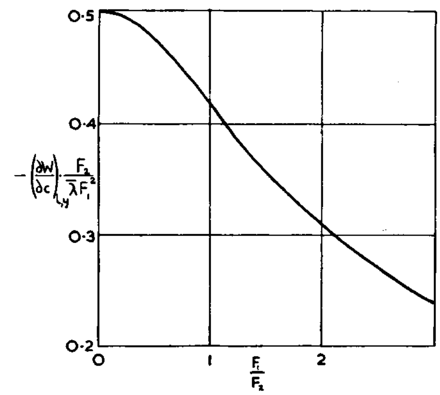

Fig. 8. Theoretical relation for split test piece from eq. (4.12).

The relation (4.12) can be conveniently represented graphically by $-(\partial W/\partial c)_{l,y}(F_2/\bar{\lambda}F_1^2)$ as a function of F_1/F_2, and this is given in Figure 8. As F_1/F_2 approaches zero, the ordinate tends to ${}^1/_2$, and, in fact, departs relatively slightly from this for values of F_1/F_2 met with in the experiments.

5. EXPERIMENTS ON THE SPLIT TEAR TEST PIECE

Consideration of this test piece suggests that a tear growing along the central line will be unstable, tending to deviate towards the edge, and experiment confirms this. To overcome this difficulty, the test pieces were scored along the intended path of the tear. By holding the rubber sheet in a special jig and using a razor blade in a holder running on guides, accurately aligned score marks could be made on both faces to a controlled depth. If the unscored thickness was about half the original, the tear was found to be stable.

Preliminary measurements were made to ensure that the elastic conditions postulated in Section 4 were fulfilled. Equation (4.5) is the crucial relation, and this could be easily checked by measuring the separation l of the grips as a function of c for various combinations of F_1 and F_2. The slope $(\partial l/\partial c)_{F_1,F_2}$ is $\lambda_1 \sin \theta$ according to eq. (4.5); θ can be found from eq. (4.9), and λ_1 from the stresses and the measured stress-strain curve. The measured value of $(\partial l/\partial c)_{F_1,F_2}$ and $\lambda_1 \sin \theta$ agreed to within 5%, provided $c/w > 3$ and $\lambda_1 > 1.15$. Both these conditions were amply fulfilled in the subsequent experiments.

The test pieces used for the tearing measurements were about 1 cm. wide, 12 cm. long, and 2 mm. thick; the unstrained cut lengths were about 5 cm. initially. The unscored thickness was found by microscopic measurement of the torn surface and was usually about 0.8 mm. Marks were made on the test piece in the unstrained state so that the rate of cut growth r could be found by timing the passage of the tip past them, as the rate must, of course, be referred to the unstrained state.

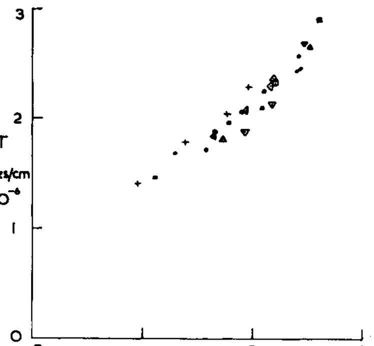

Fig. 9. Comparison of results from simple extension and split tear test pieces: (●) simple extension; split test piece with (⊡) F_2 = 0.7 kg., ( ) F_2 = 0.6 kg., (+) F_2 = 0.5 kg., (△) F_2 = 0.4 kg., (▽) F_2 = 0.3 kg.

The test pieces were loaded under various combinations of F_1 and F_2 and the corresponding rates of tearing measured. These were always below the critical rate r_c ($\sim 10^{-2}$ cm./sec.), so that the tearing was steady. The values of T were calculated from Figure 8 and eq. (2.1), $\bar{\lambda}$ being found from the measured stress–strain curve of the material. In eq. (2.1), the torn thickness must now be used.

In order to check the tear criterion, the above results were compared with those from simple extension test pieces. It was necessary for these to be scored in a similar manner, as it was possible that this might affect the tearing process. The simple extension test pieces were cut from the same sheet and tested as described in Section 3. The results from both test pieces are shown in Figure 9, correction having been made for slight variations in ambient temperature from 20°C. The agreement is again very satisfactory.

Although the above comparison was made with the use of scored test pieces throughout for consistency, the T-r relation obtained differs in fact very little from that found for the unscored ones (Fig. 6).

The results from the three tear test pieces at low tearing rates are all mutually consistent when expressed in terms of the tearing energy, despite their very different shapes. This agreement is good evidence for the fundamental nature of this parameter.

The discrepancy at high tearing rates found initially between the pure shear and simple extension results could be largely eliminated by applying the tearing force to the latter test piece so that fluctuations in rate and tearing force were minimized. This quite marked effect on the results from this test piece of the method of applying the load indicates that the significance of such measurements must be assessed cautiously if there is any tendency for stick-slip behavior. In part III, stick-slip tearing was ascribed to the formation of a strengthening structure at low rates of tear that had insufficient time to form at high rates, giving a negative slope to the T-r relation. This explanation could also hold for the exaggerated effect found here, although the magnitude is rather surprising for a material which is supposed to be noncrystallizing. The change in the appearance of the torn surface from rough to smooth at the critical tearing rate r_c suggests that the change in T is associated primarily with a change in the effective radius of the tear tip.[2]

The validity of the tearing energy concept has now been tested under a variety of conditions: (*1*) for catastrophic tearing of natural rubber,[1] (*2*) for cut growth in natural rubber,[5] and (*3*) steady tearing of GR-S. The same concept appears to be valid for the hard plastics, polymethyl methacrylate and polystyrene (Benbow and Roesler[8]). The original work along these lines by Griffith[9] was on glass. The theory, therefore, appears to be of quite wide application.

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Synopsis

A criterion for the tearing of rubber based on an energy balance approach, proposed previously, has been critically examined experimentally. This criterion implies that the energy required to form unit area of surface by tearing should be a constant of the material, and it can be tested by finding if tearing results from test pieces of different shapes are self-consistent. Previous work has indicated that the criterion is approximately correct for natural rubber, and the present paper gives a much more rigorous check for a noncrystallizing and therefore more convenient material (GR-S). The results from three test pieces of widely different shapes show excellent agreement. The material used exhibited an abrupt change in the mode of tearing as the tearing energy increased through 3×10^6 ergs/cm.^{2}, the rate of tearing suddenly increasing from about 10^{-2} to 10 cm./sec. This correlated with a change in the appearance of the torn surfaces from rough to smooth.

Résumé

Un critère pour l'étirement du caoutchouc basé sur une balance d'énergie, proposé précédemment, a été vérifié expérimentalement. Ce critère implique que l'énergie nécessaire pour former une surface unitaire par étirement, est une constante de la substance; cela peut être, controlé en vérifiant si les résultats d'étirement d'échantillons de différentes formes se confirment d'un l'autre. Des travaux précédents ont indiqué que le critère est approximativement correct pour le caoutchouc naturel, et ce compte rendu ci donne un contrôle beaucoup plus rigoureux, utilisant une substance non cristallisable, et par conséquent plus commode (GR-S). Les résultats de trois échantillons de substance très différents montrent un accord excellent. La substance employée montre un changement brusque dans le mode d'étirement quand l'énergie d'étirement passe par 3×10^6 ergs/cm^2 la vitesse d'étirement croîssant brusquement d'environ 10^{-2} à 10 cm/sec. Ceci correspond à un changement dans l'aspect de la surface d'étirement, passant de rugeuse à lisse.

Zusammenfassung

Ein schon früher vorgeschlagenes Zerreisskriterium für Kautschuk, das auf einer Energiebilanz beruht, wurde einer kritischen, experimentellen Prüfung unterzogen. Dieses Kriterium beinhaltet die Aussage, dass die zur Bildung der Flächeneinheit der Oberfläche durch Zerreissen erforderliche Energie eine Materialkonstante sein sollte und es kann daher an Befunden überprüft werden, die zeigen ob Zerreissversuche an Testproben von verschiedener Gestalt übereinstimmende Ergebnisse liefern. Frühere Versuche haben gezeigt, dass das Kriterium bei Naturkautschuk annähernd korrekt ist und die vorliegende Mitteilung bringt eine noch viel strengere Prüfung an einem nichtkristallisierenden und daher besser zu handhabenden Material (GR-S). Die Ergebnisse an drei Testproben mit stark unterschiedlicher Gestalt zeigen ausgezeichnete Übereinstimmung. Das verwendete Material liess beim Anwachsen der Zerreissenergie über 3×10^6 erg/cm^2 eine abrupte Änderung des Zerreissvorganges erkennen, indem die Zerreissgeschwindigkeit plötzlich von etwa 10^{-2} auf 10 cm/sec anstieg. Diesem Übergang entsprach eine Änderung des Aussehens der gerissenen Oberflächen und zwar von einem rauhen zu einem glatten Zustand.

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